

Quiz 7 Sample

Question 1. (10 pts)

Let

$$A = \begin{bmatrix} | & | & | & | & | \\ v_1 & v_2 & v_3 & v_4 & v_5 \\ | & | & | & | & | \end{bmatrix}$$

where v_1, v_2, \dots, v_5 are linearly independent vectors in \mathbb{R}^5 .

- (a) Determine whether
- $\det(A)$
- is zero or not.

Solution: $\det(A) \neq 0$, since v_1, v_2, \dots, v_5 are linearly independent.

- (b) Write the determinant of each following matrix in terms of
- $\det(A)$
- .

Solution:

$$(a) \det \begin{bmatrix} | & | & | & | & | \\ (v_1 + 2v_3) & v_2 & v_3 & v_4 & v_5 \\ | & | & | & | & | \end{bmatrix} = \det(A)$$

$$(b) \det \begin{bmatrix} | & | & | & | & | \\ v_1 & v_4 & v_3 & (v_2 + v_5) & v_5 \\ | & | & | & | & | \end{bmatrix} = (-1) \det(A)$$

$$(c) \det \left(k \begin{bmatrix} | & | & | & | & | \\ v_1 & v_2 & v_3 & v_4 & v_5 \\ | & | & | & | & | \end{bmatrix} \right) = k^5 \det(A)$$

$$(d) \det \begin{bmatrix} | & | & | & | & | \\ v_1 & v_2 & (kv_3) & v_4 & v_5 \\ | & | & | & | & | \end{bmatrix} = k \det(A)$$

Note the difference between part (c) and (d).

Question 2. (10 pts)

- (a) Find all eigenvalues and eigenvectors of $B = \begin{bmatrix} 3 & -1 & 1 \\ 7 & -5 & 1 \\ 6 & -6 & 2 \end{bmatrix}$.
- (b) Determine whether B is diagonalizable.

Solution: This is Exercise 9.15 on Page 313 of the textbook.